Electronic Circuits /2/

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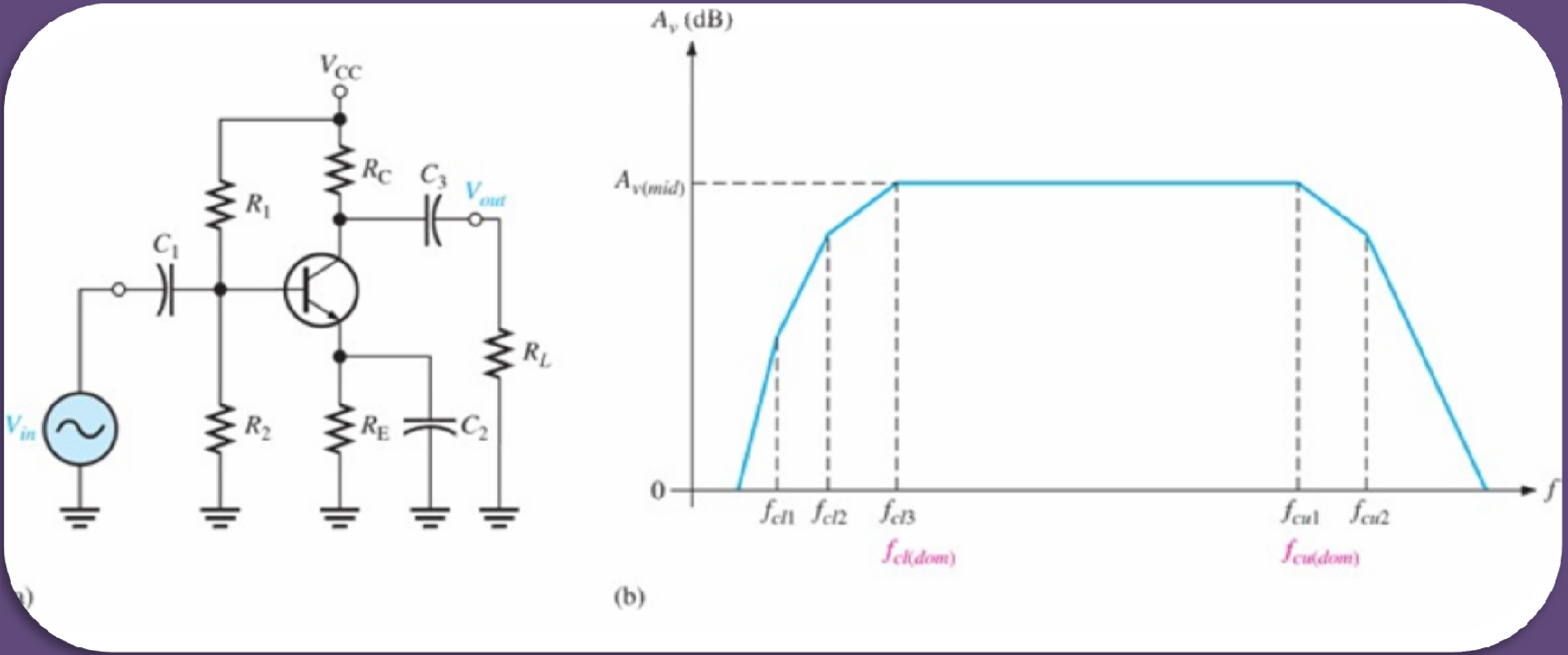
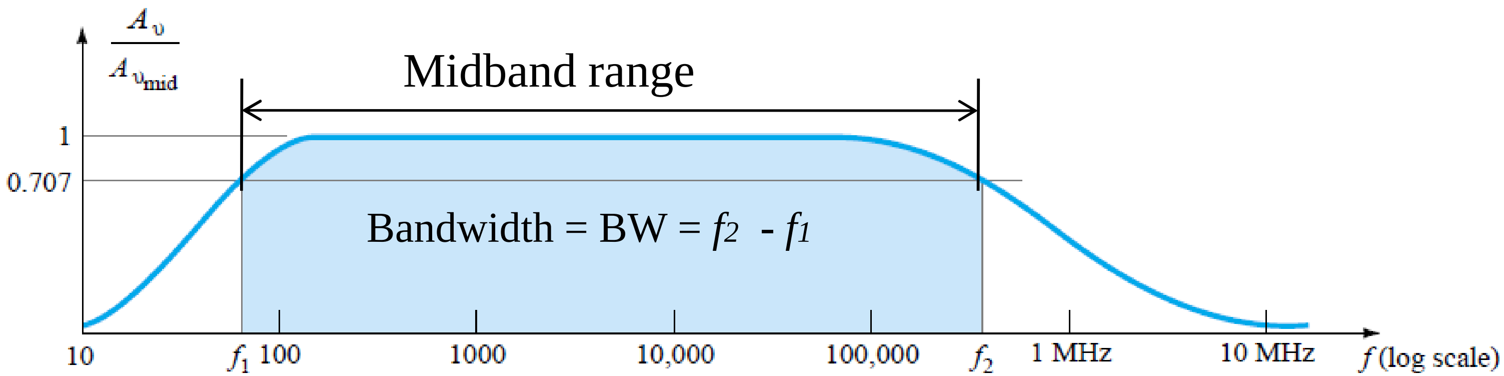
CHAPTER /3/

FREQUENCY RESPONSE OF AMPLIFIERS



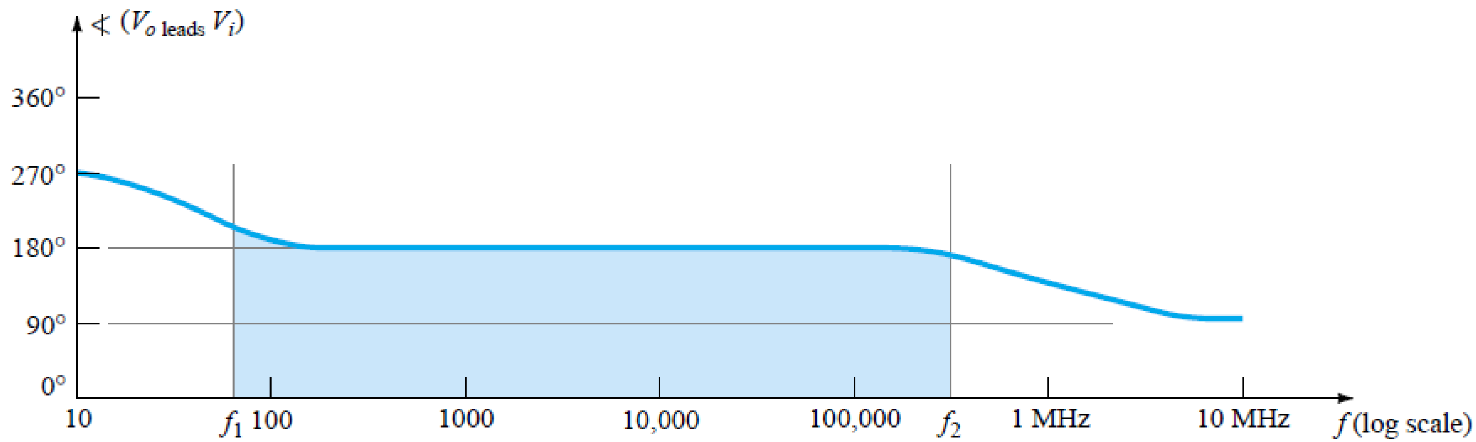
1. Introduction

Frequency Response of an amplifier is the change in gain or phase shift over a specified range of input signal frequency



f_1 : lower cutoff frequency

f_2 : upper cutoff frequency



2. The Decibel

The concept of the decibel (dB) and the associated calculations will become increasingly important in the remaining sections of this chapter. The background surrounding the term *decibel* has its origin in the established fact that power and audio levels are related on a logarithmic basis. That is, an increase in power level, say 4 to 16 W, does not result in an audio level increase by a factor of 16/4 = 4. It will increase by a factor of 2 as derived from the power of 4 in the following manner: $(4)^2 = 16$. For a change of 4 to 64 W, the audio level will increase by a factor of 3 since $(4)^3 = 64$. In logarithmic form, the relationship can be written as $\log_4 64 = 3$.

Power gain is expressed in decibel (DB) by the following formula:

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} \qquad \text{dB}$$

Voltage gain is expressed in decibel by the following formula:

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_i}{V_1^2/R_i} = 10 \log_{10}\left(\frac{V_2}{V_1}\right)^2$$

$$G_{dB} = 20 \log_{10} \frac{V_2}{V_1} \qquad \text{dB}$$

Example 1

The input power to a device is 10,000 W at a voltage of 1000 V. The output power is 500 W, while the output impedance is 20 Ω.

(a) Find the power gain in decibels.

(b) Find the voltage gain in decibels.

(c) Explain why parts (a) and (b) agree or disagree.

Solution

Example 2

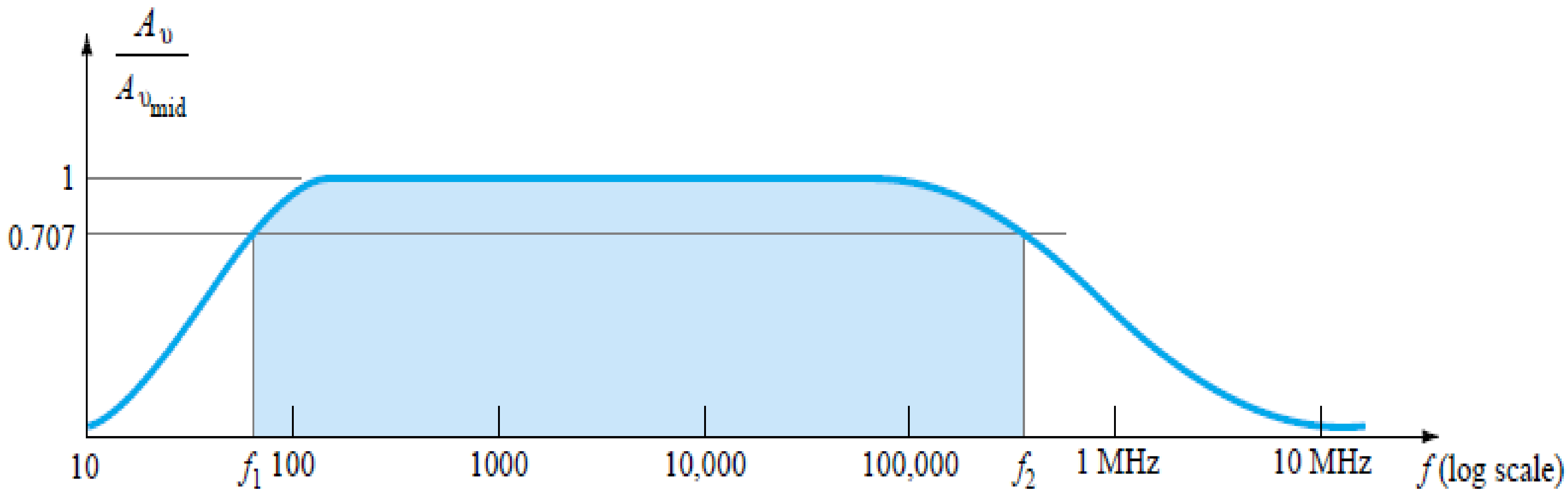
An amplifier rated at 40-W output is connected to a 10-Ω speaker.

(a) Calculate the input power required for full power output if the power gain is 25 dB.

(b) Calculate the input voltage for rated output if the amplifier voltage gain is 40 dB.

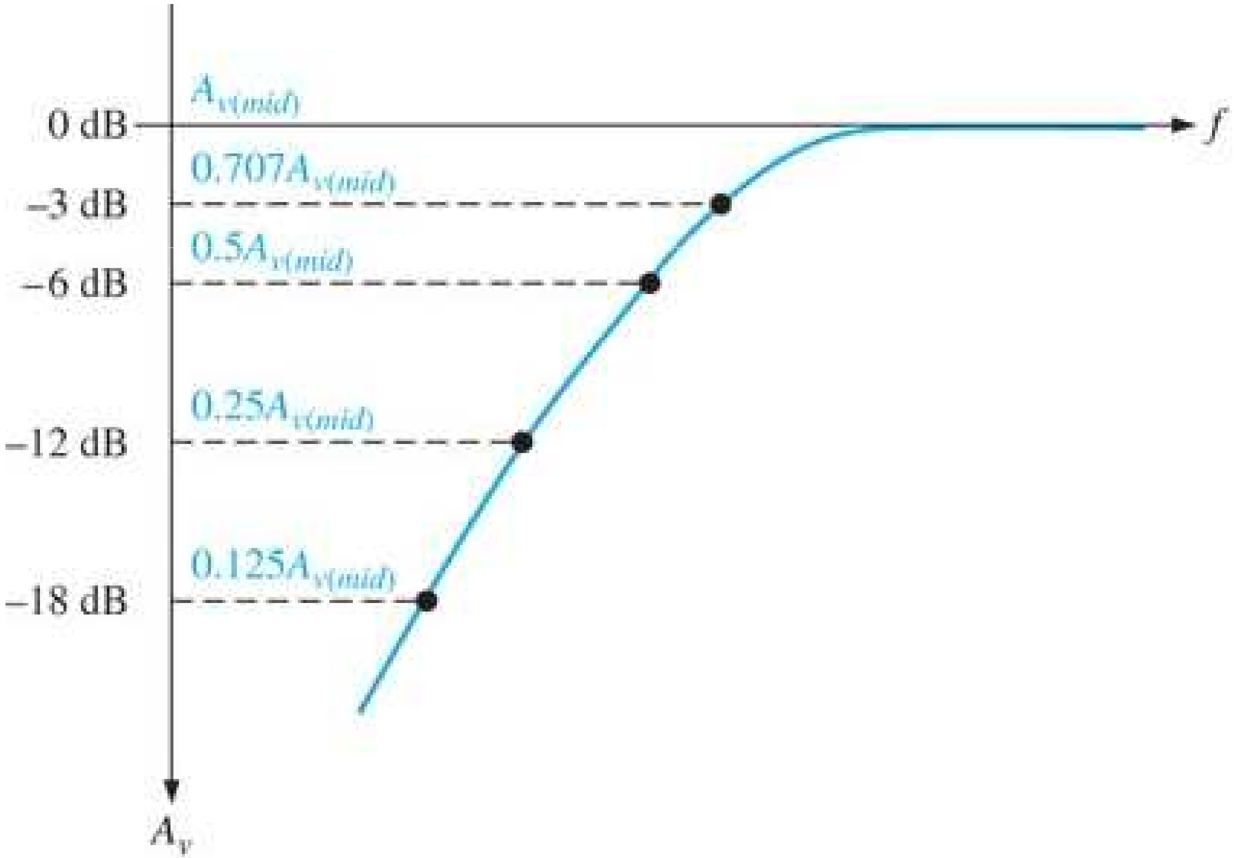
Solution

For more applications a communications nature (audio, video), a decibel plot of the voltage gain versus frequency is more used. Before obtaining the logarithmic plot, however, the curve is generally normalized as shown below. In this figure, the gain at each frequency is divided by the midband value, the midband value is then 1 as indicated. At the half-power frequency, the resulting level is 0.707 = .



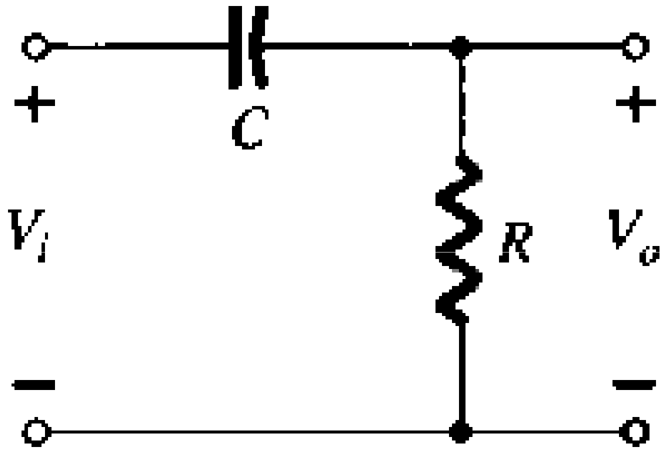
A decibel plot can now be obtained by applying the following equation:

$$\left.\frac{A_v}{A_{v_{\text{mid}}}}\right|_{\text{dB}} = 20\log_{10}\frac{A_v}{A_{v_{\text{mid}}}}$$



3. Low-Frequency Analysis BODE Plot

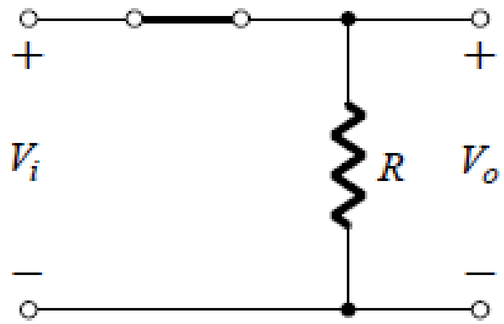
In the low-frequency region of the single-stage BJT or FET amplifier, it is the *R*-*C* combinations formed by the network capacitors C_C, C_E, and C_s and the network resistive parameters that determine the cutoff frequencies.



Our analysis, therefore, will begin with the series *R*-*C* combination of Fig. 11.8 and the development of a procedure that will result in a plot of the frequency response with a minimum of time and effort.

At very high frequencies,

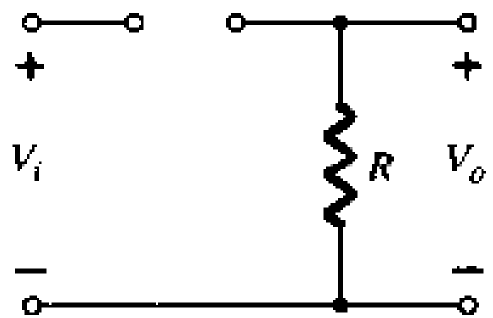
$$X_C = \frac{1}{2\pi f C} \cong 0\ \Omega$$



and the short-circuit equivalent can be substituted for the capacitor. The result is that $V_o \cong V_i$ at high frequencies.

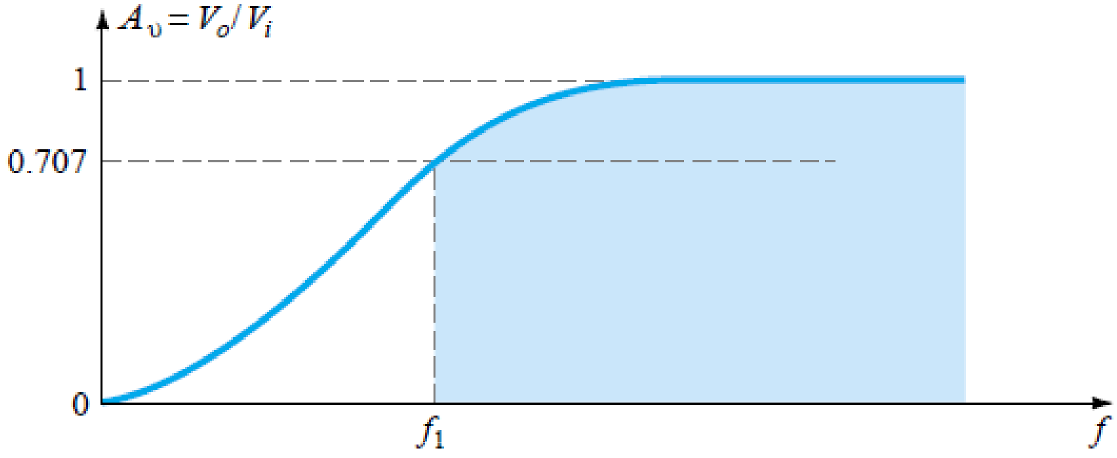
At $f = 0$ Hz,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(0)C} = \infty\ \Omega$$



and the open-circuit approximation can be applied with the result that $V_o = 0$ V.

Between the two extremes, the ratio $A_v = V_o/V_i$ will vary as shown below. As the frequency increases, the capacitive reactance decreases and more of the input voltage appears across the output terminals.



The output and input voltages are related by the voltage-divider rule in the following manner:

$$\mathbf{V}_o = \frac{\mathbf{R}\mathbf{V}_i}{\mathbf{R} + \mathbf{X}_C}$$

with the magnitude of V_o determined by

$$V_o = \frac{RV_i}{\sqrt{R^2 + X_C^2}}$$

For the special case where $X_C = R$,

$$V_o = \frac{RV_i}{\sqrt{R^2X_C^2}} = \frac{RV_i}{} = \frac{RV_i}{\sqrt{2R^2}} = \frac{RV_i}{\sqrt{2}R} = \frac{1}{\sqrt{2}}\,V_i$$

and

$$|A_v| = \frac{V_o}{V_i} = \frac{1}{\sqrt{2}} = 0.707\big|_{X_C=R}$$

The frequency at which this occurs is determined from

$$X_C = \frac{1}{2\pi f_1 C} = R$$

and

$$f_1 = \frac{1}{2\pi RC}$$

In terms of logs,

$$G_v = 20\log_{10} A_v = 20\log_{10}\frac{1}{\sqrt{2}} = -3\ \text{dB}$$

while at $A_v = V_o/V_i = 1$ or $V_o = V_i$ (the maximum value),

$$G_v = 20 \log_{10} 1 = 20(0) = 0 \text{ dB}$$

- If the gain equation is written as

$$A_v = \frac{V_o}{V_i} = \frac{R}{R - jX_C} = \frac{1}{1 - j(X_C/R)} = \frac{1}{1 - j(1/\omega CR)} = \frac{1}{1 - j(1/2\pi fCR)}$$

and using the frequency defined above,

$$\boxed{A_v = \frac{1}{1 - j(f_1/f)}}$$

In the magnitude and phase form,

$$A_v = \frac{V_o}{V_i} = \underbrace{\frac{1}{\sqrt{1 + (f_1/f)^2}}}_{\text{magnitude of } A_v} \underbrace{\angle \tan^{-1}(f_1/f)}_{\text{phase } \sphericalangle \text{ by which } V_o \text{ leads } V_i}$$

In the logarithmic form, the gain in dB is

$$A_{v(\mathrm{dB})} = -20 \log_{10} \frac{f_1}{f}$$

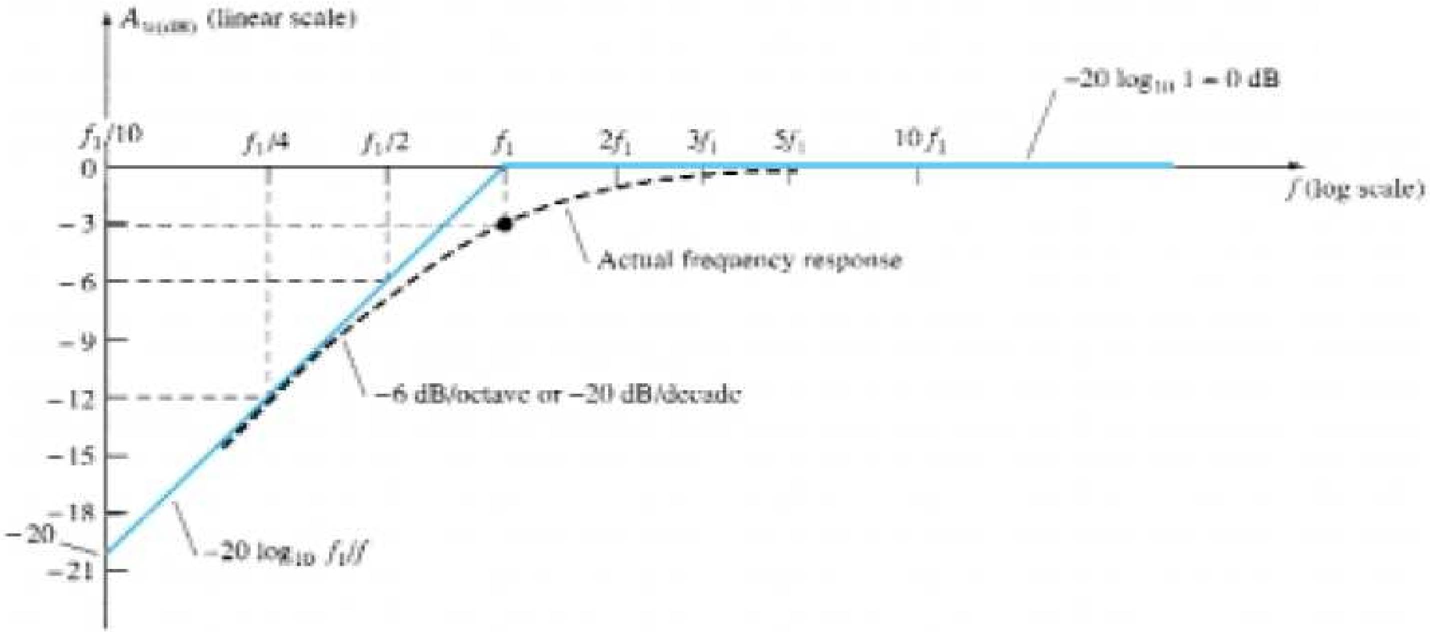
At $f = f_1$: $\frac{f_1}{f} = 1$ and $-20 \log_{10} 1 = 0$ dB

At $f = \frac{1}{2} f_1$: $\frac{f_1}{f} = 2$ and $-20 \log_{10} 2 \cong -6$ dB

At $f = \frac{1}{4} f_1$: $\frac{f_1}{f} = 4$ and $-20 \log_{10} 4 \cong -12$ dB

At $f = \frac{1}{10} f_1$: $\frac{f_1}{f} = 10$ and $-20 \log_{10} 10 = -20$ dB

A plot of these points is indicated below.



The calculations above and the curve itself demonstrate clearly that:

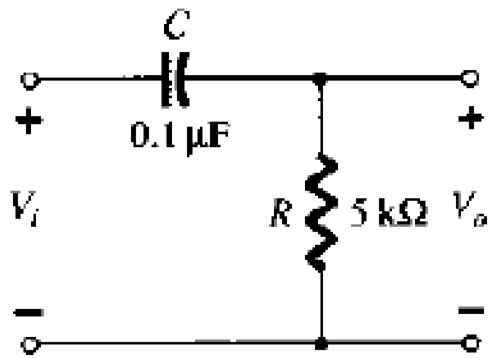
A change in frequency by a factor of 2, equivalent to 1 octave, results in a 6-dB change in the ratio as noted by the change in gain from $f_1/2$ to f_1.

As noted by the change in gain from $f_1/2$ to f_1:

For a 10:1 change in frequency, equivalent to 1 decade, there is a 20-dB change in the ratio as demonstrated between the frequencies of $f_1/10$ and f_1.

Example 3

For the network
(a) Determine the break frequency.
(b) Sketch the asymptotes and locate the −3-dB point.
(c) Sketch the frequency response curve.



Solution

The gain at any frequency can then be determined from the frequency plot in the following manner:

$$A_{v(\mathrm{dB})} = 20\log_{10}\frac{V_o}{V_i}$$

but
$$\frac{A_{v(\mathrm{dB})}}{20} = \log_{10}\frac{V_o}{V_i}$$

and
$$\boxed{A_v = \frac{V_o}{V_i} = 10^{\left(\frac{A_{v(\mathrm{dB})}}{20}\right)}}$$

For example, if $A_{v(\mathrm{dB})} = -3$ dB,

$$A_v = \frac{V_o}{V_i} = 10^{(-3/20)} = 10^{(-0.15)} \cong 0.707 \qquad \text{as expected}$$

$A_{v(\mathrm{dB})} \cong -1$ dB at $f = 2f_1 = 637$ Hz. The gain at this point is

$$A_v = \frac{V_o}{V_i} = 10^{\left(\frac{A_{v(\mathrm{dB})}}{20}\right)} = 10^{(-1/20)} = 10^{(-0.05)} = 0.891 \Longrightarrow V_o = 0.891 V_i$$

The phase angle of θ is determined from

$$\theta = \tan^{-1}\frac{f_1}{f}$$

For frequencies $f \ll f_1$,

$$\theta = \tan^{-1}\frac{f_1}{f} \to 90^\circ$$

For instance, if $f_1 = 100f$,

$$\theta = \tan^{-1}\frac{f_1}{f} = \tan^{-1}(100) = 89.4^\circ$$

For $f = f_1$,

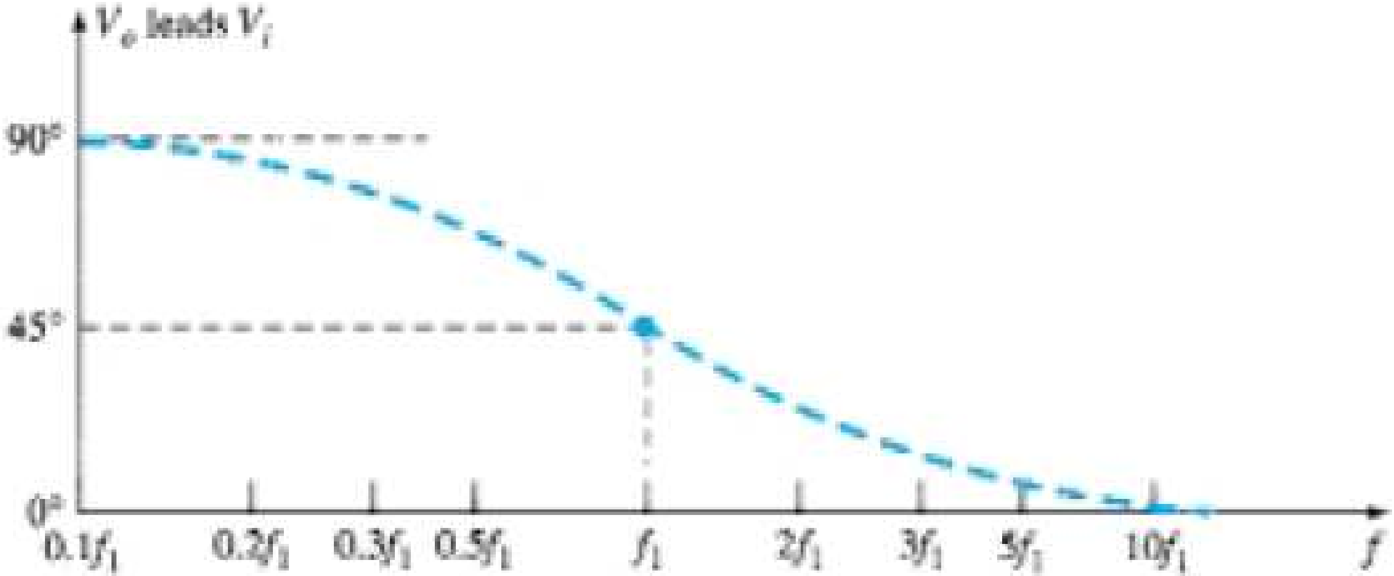
$$\theta = \tan^{-1}\frac{f_1}{f} = \tan^{-1}1 = 45^\circ$$

For $f \gg f_1$,

$$\theta = \tan^{-1} \frac{f_1}{f} \rightarrow 0°$$

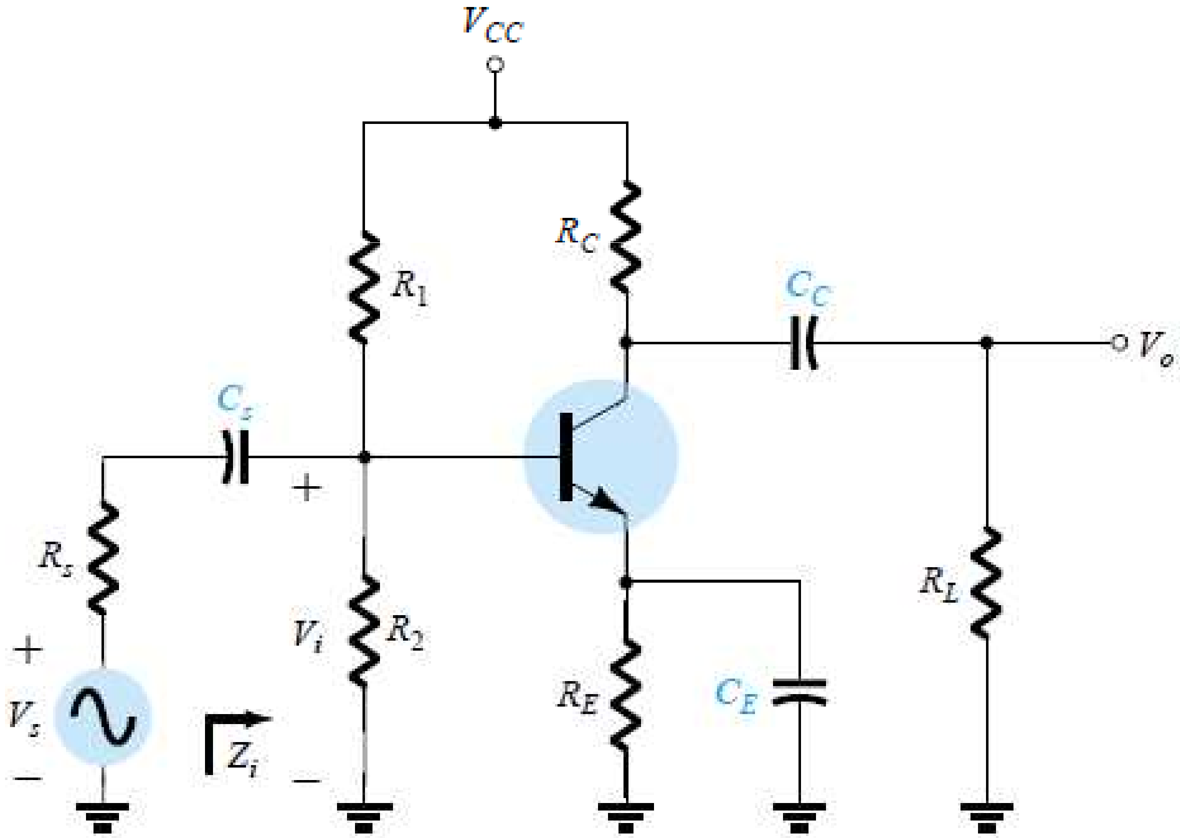
For instance, if $f = 100f_1$,

$$\theta = \tan^{-1} \frac{f_1}{f} = \tan^{-1} 0.01 = 0.573°$$



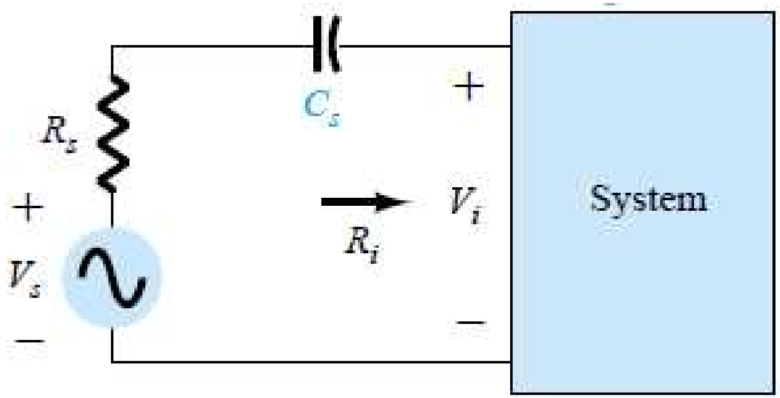
4. Low-Frequency Response – BJT Amplifier

The analysis of this section will employ the loaded voltage-divider BJT bias configuration, but the results can be applied to any BJT configuration. It will simply be necessary to find the appropriate equivalent resistance for the R-C combination. For the network below , the capacitors C_s, C_C, and C_E will determine the low-frequency response. We will now examine the impact of each independently in the order listed.



C_s

Since C_s is normally connected between the applied source and the active device, the general form of the R-C configuration is established by the network below.



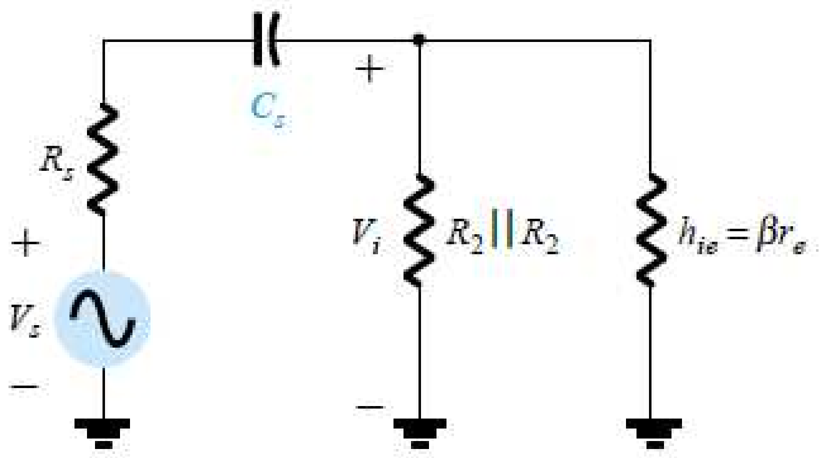
The cutoff frequency is,

$$f_{LS} = \frac{1}{2\pi(R_S + R_i)C_S}$$

The voltage *Vi* will then be related to *Vs* by,

$$V_i = V_S \frac{R_i}{R_S + R_i}$$

The value of R_i is determined by



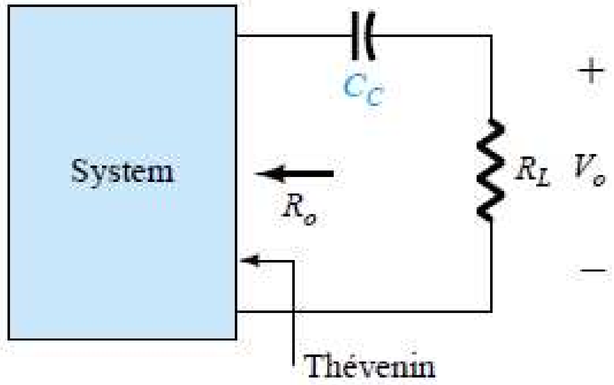
$$R_i = R_1 || R_2 || \beta r_e$$

The voltage $\mathbf{V}_i$ applied to the input of the active device can be calculated using the voltage-divider rule:

$$V_i = V_S \frac{R_i}{R_S + R_i + jX_{Cs}}$$

C_C

Since the coupling capacitor is normally connected between the output of the active device and the applied load, the *R*-*C* configuration that determines the low cutoff frequency due to C_C appears

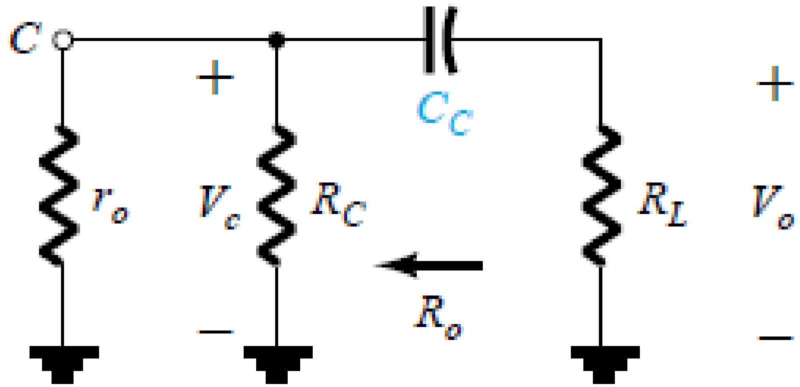


The cutoff frequency due to Cc is determined by:

$$f_{LS} = \frac{1}{2\pi(R_L + R_O)C_C}$$

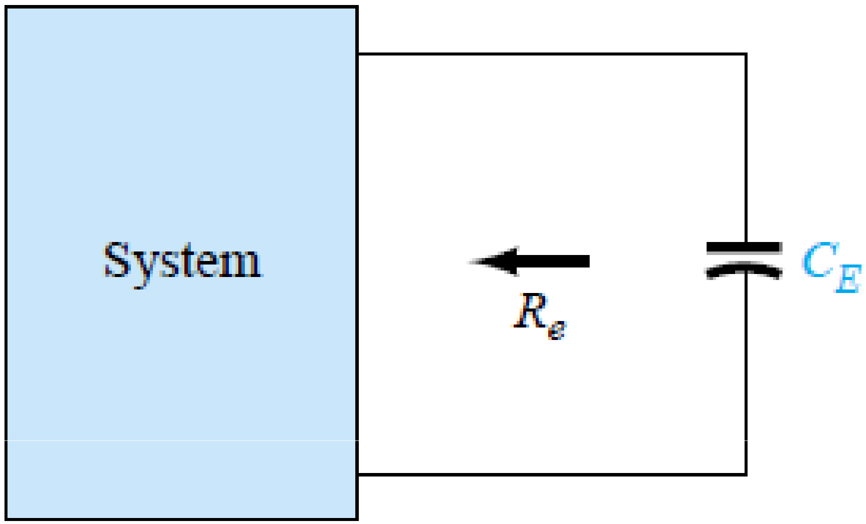
The ac equivalent network for the output section with *Vi* = 0 V.

$$R_o = R_C \| r_o$$



C_E

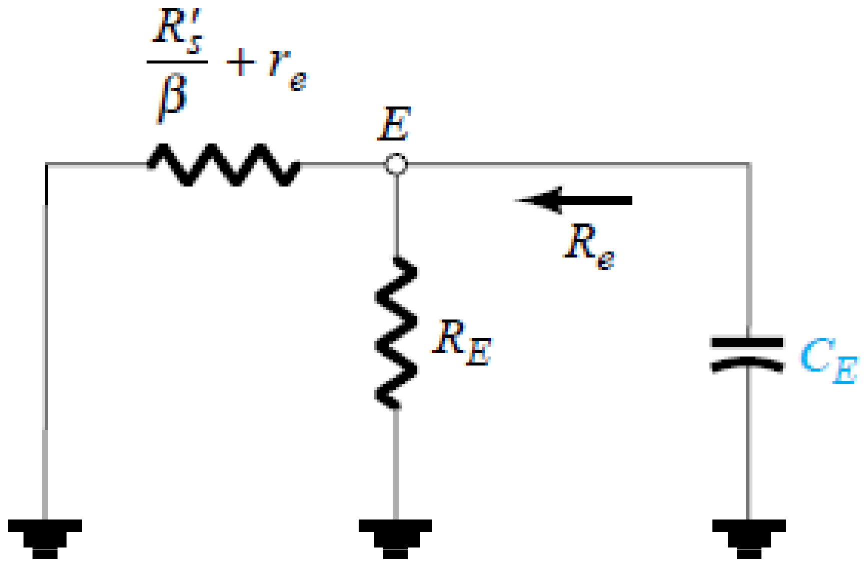
To determine f_{L_E}, the network "seen" by C_E must be determined below.



Once the level of R_e is established, the cutoff frequency due to C_E can be determined using the following equation:

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

The ac equivalent network as ‘seen’ by CE appears in following Fig

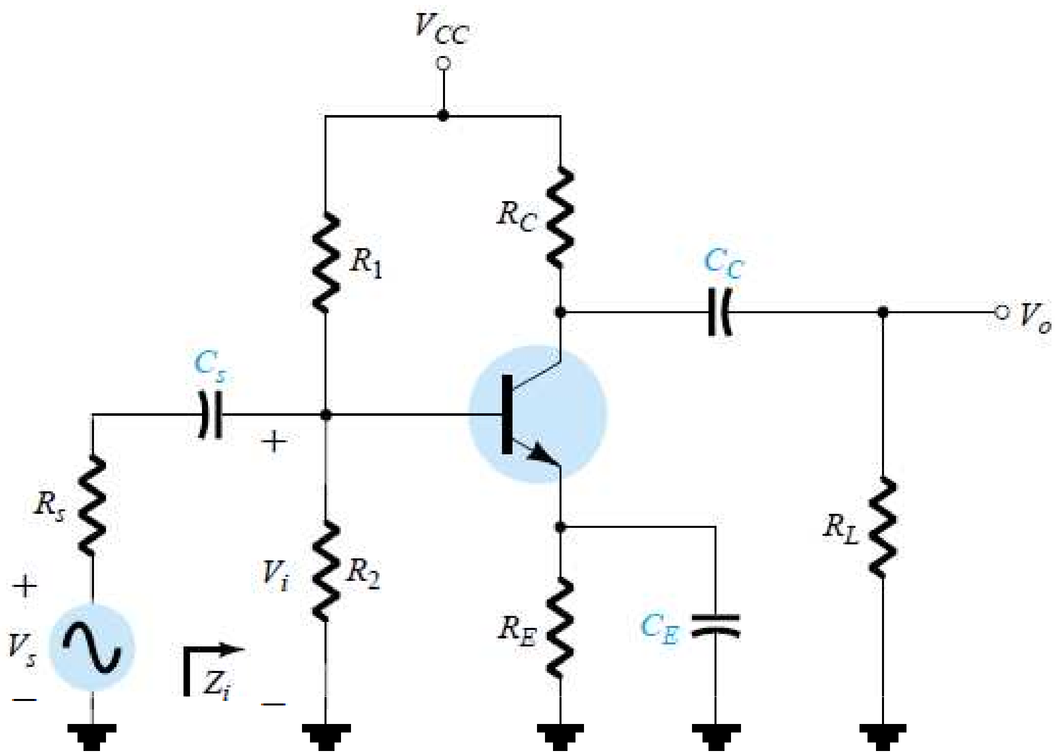


The value of Re is therefore determined by

$$R_e = R_E \| \left(\frac{R'_s}{\beta} + r_e \right)$$

where $R'_s = R_s \| R_1 \| R_2$.

Example 4



(a) Determine the lower cutoff frequency for the following network using the following parameters:

$C_s = 10\ \mu F$, $C_E = 20\ \mu F$, $C_C = 1\ \mu F$

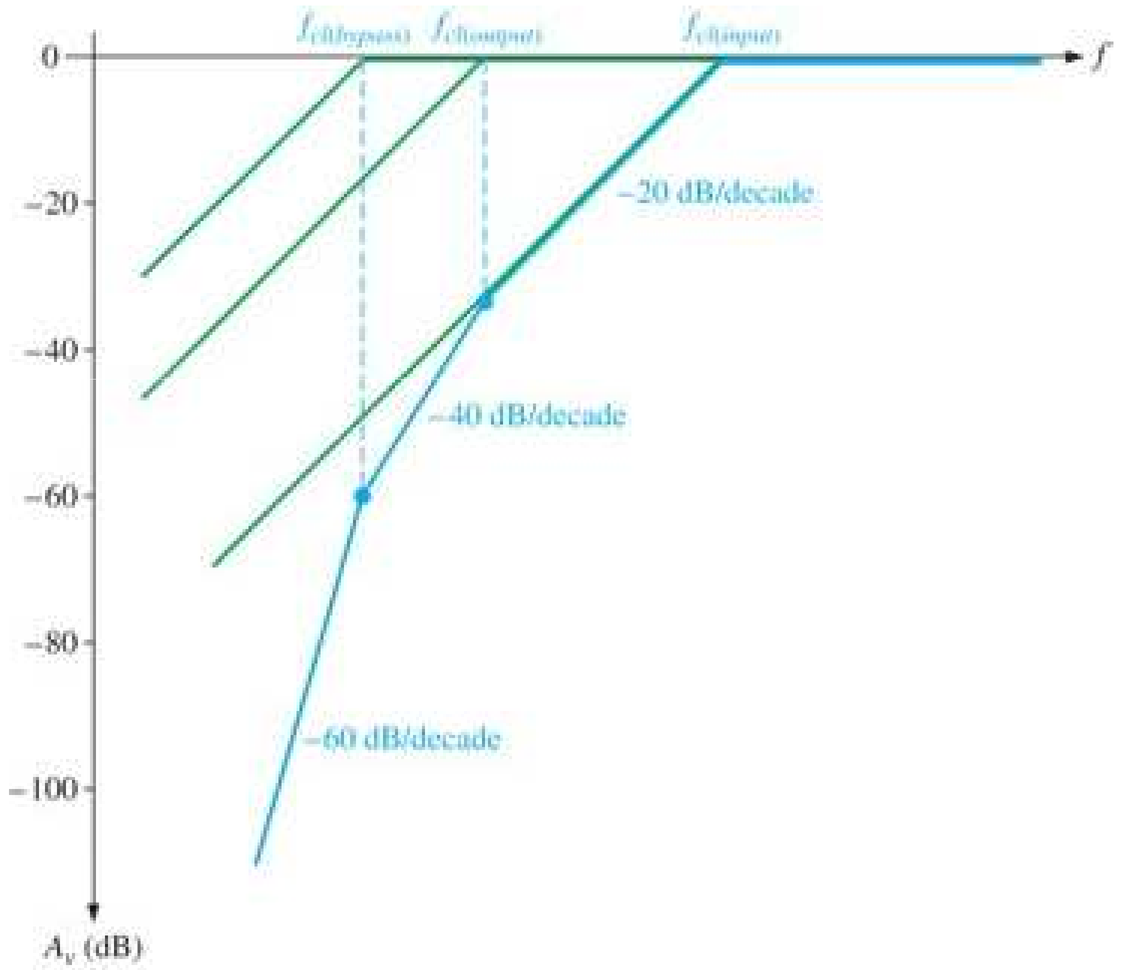
$R_s = 1\ k\Omega$, $R_1 = 40\ k\Omega$, $R_2 = 10\ k\Omega$, $R_E = 2\ k\Omega$, $R_C = 4\ k\Omega$, $R_L = 2.2\ k\Omega$

$\beta = 100$, $r_o = \infty\ \Omega$, $V_{CC} = 20\ V$

(b) Sketch the frequency response using a Bode plot.

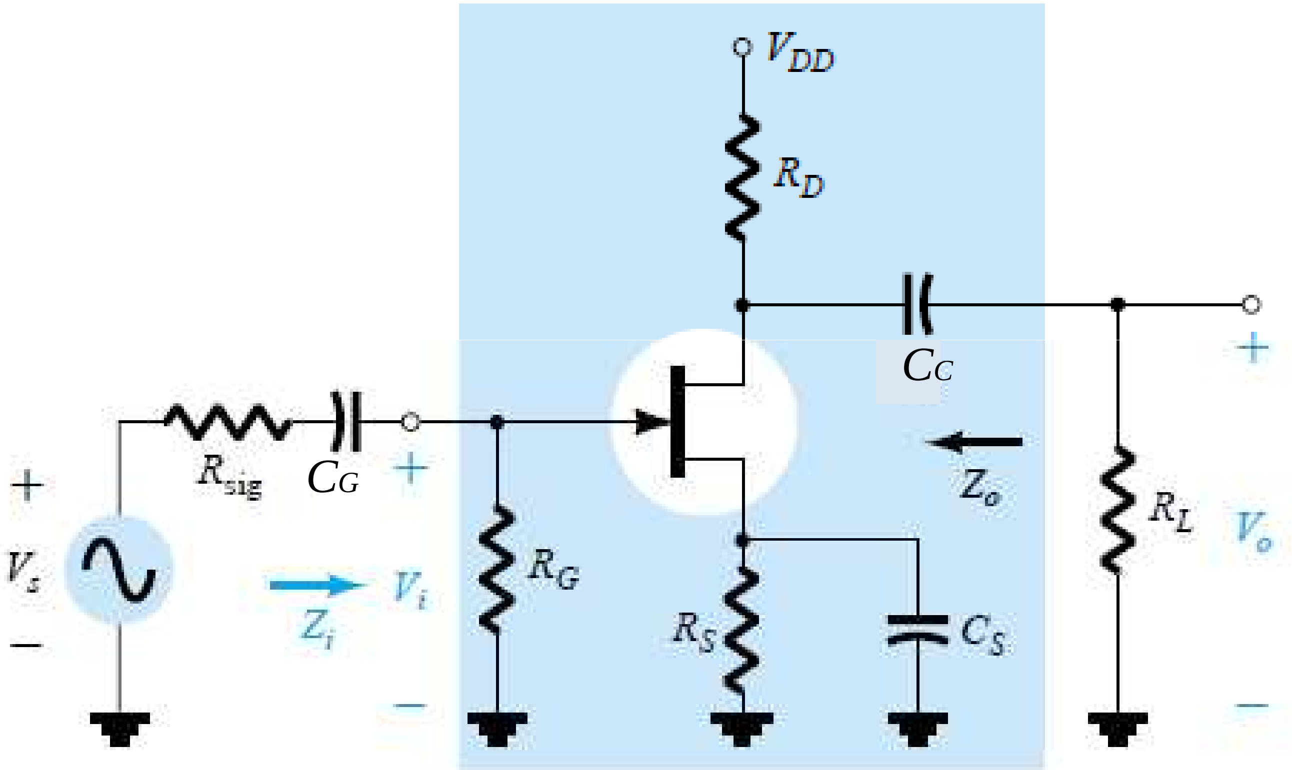
Solution

Composite Bode plot of a BJT amplifier response for three low-frequency *RC* circuits with different critical frequencies. Total response is shown by the blue curve.



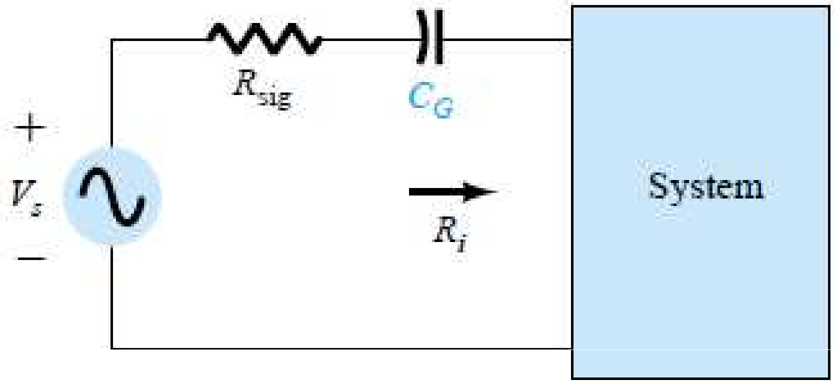
4. Low-Frequency Response – FET Amplifier

The analysis of the FET amplifier in the low-frequency region will be quite similar to that of the BJT amplifier



C_G

For the coupling capacitor between the source and the active device, the ac equivalent network will appear as :the following figure:



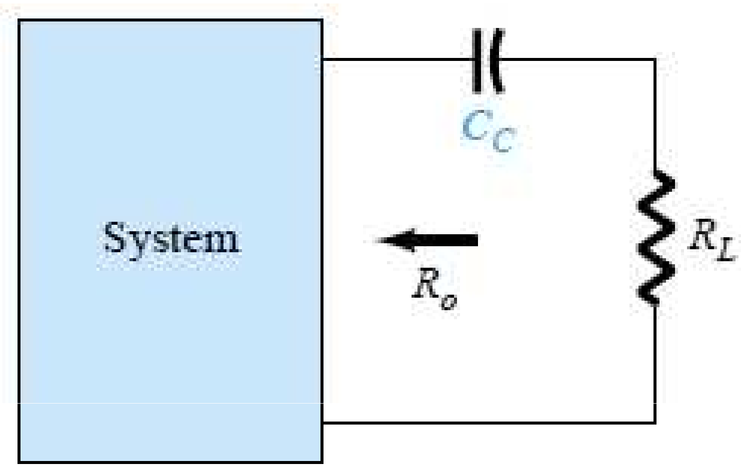
The cutoff frequency determined by *C*G will then be:

$$f_{L_G} = \frac{1}{2\pi(R_{sig} + R_i)C_G}$$

$$R_i = R_G$$

C_C

For the coupling capacitor between the active device and the load the following network will result.



The resulting cutoff frequency is:

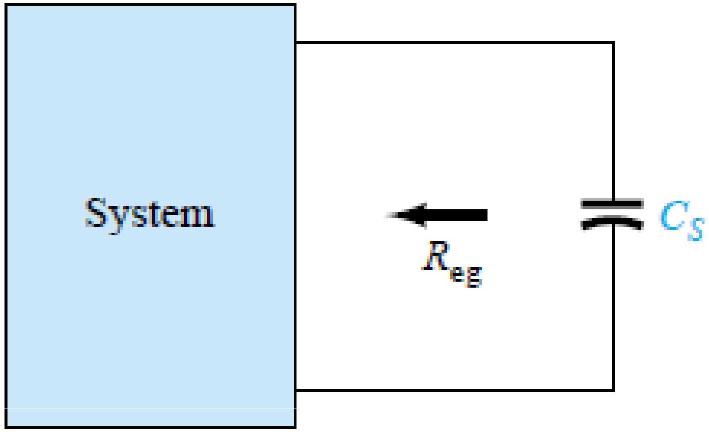
$$f_{L_C} = \frac{1}{2\pi(R_o + R_L)C_C}$$

$$R_o = R_D \| r_d$$

C_S

For the source capacitor C_S, the resistance level of importance is defined by

llowing network

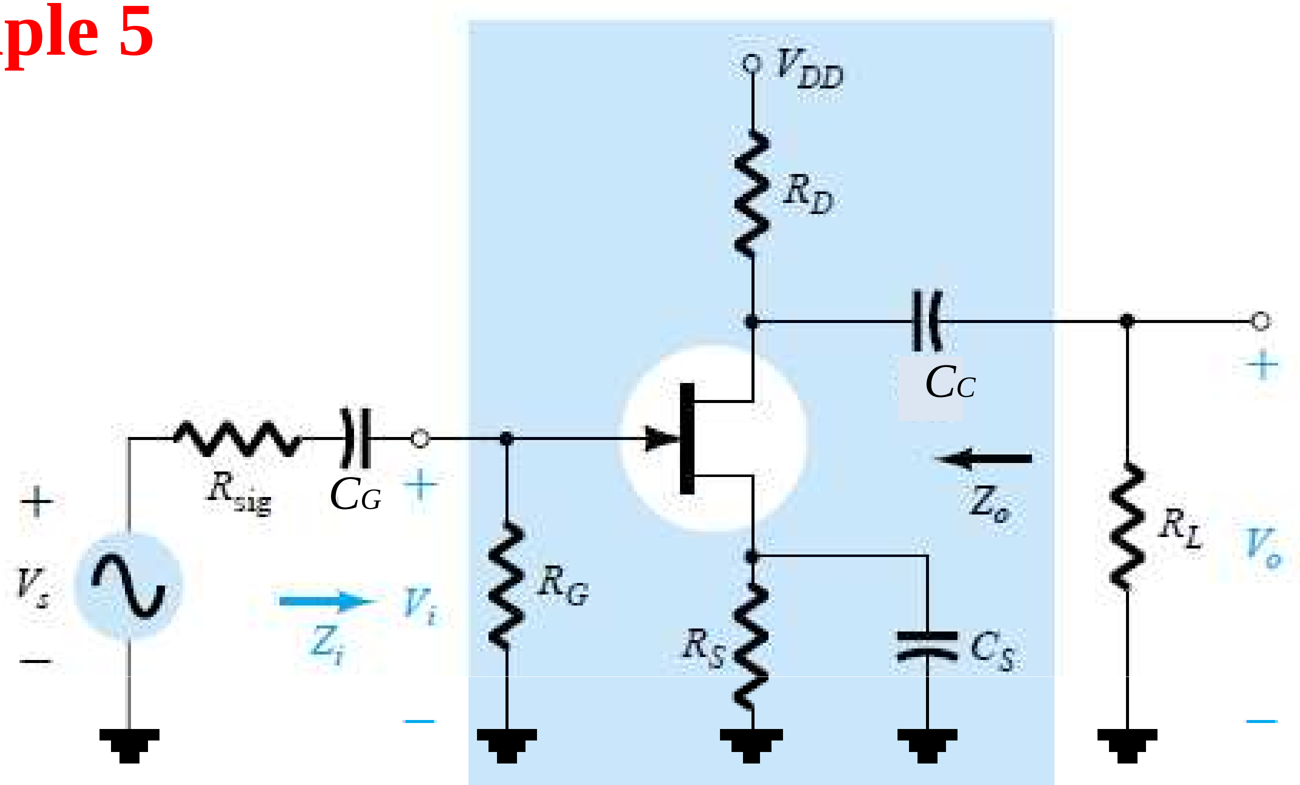


The cutoff frequency will be defined by

$$f_{L_S} = \frac{1}{2\pi R_{eq} C_S}$$

$$R_{eq} = \frac{R_S}{1 + R_S(1 + g_m r_d)/(r_d + R_D \| R_L)}$$

Example 5



(a) Determine the lower cutoff frequency for the following network using the following parameters:

$C_G = 0.01\ \mu F$, $C_C = 0.5\ \mu F$, $C_S = 2\ \mu F$

$R_{sig} = 10\ k\Omega$, $R_G = 1\ M\Omega$, $R_D = 4.7\ k\Omega$, $R_S = 1\ k\Omega$, $R_L = 2.2\ k\Omega$

$I_{DSS} = 8mA$, $V_P = -4\ V$ $r_d = \infty\ \Omega$, $V_{DD} = 20\ V$

(b) Sketch the frequency response using a Bode plot.

Solution

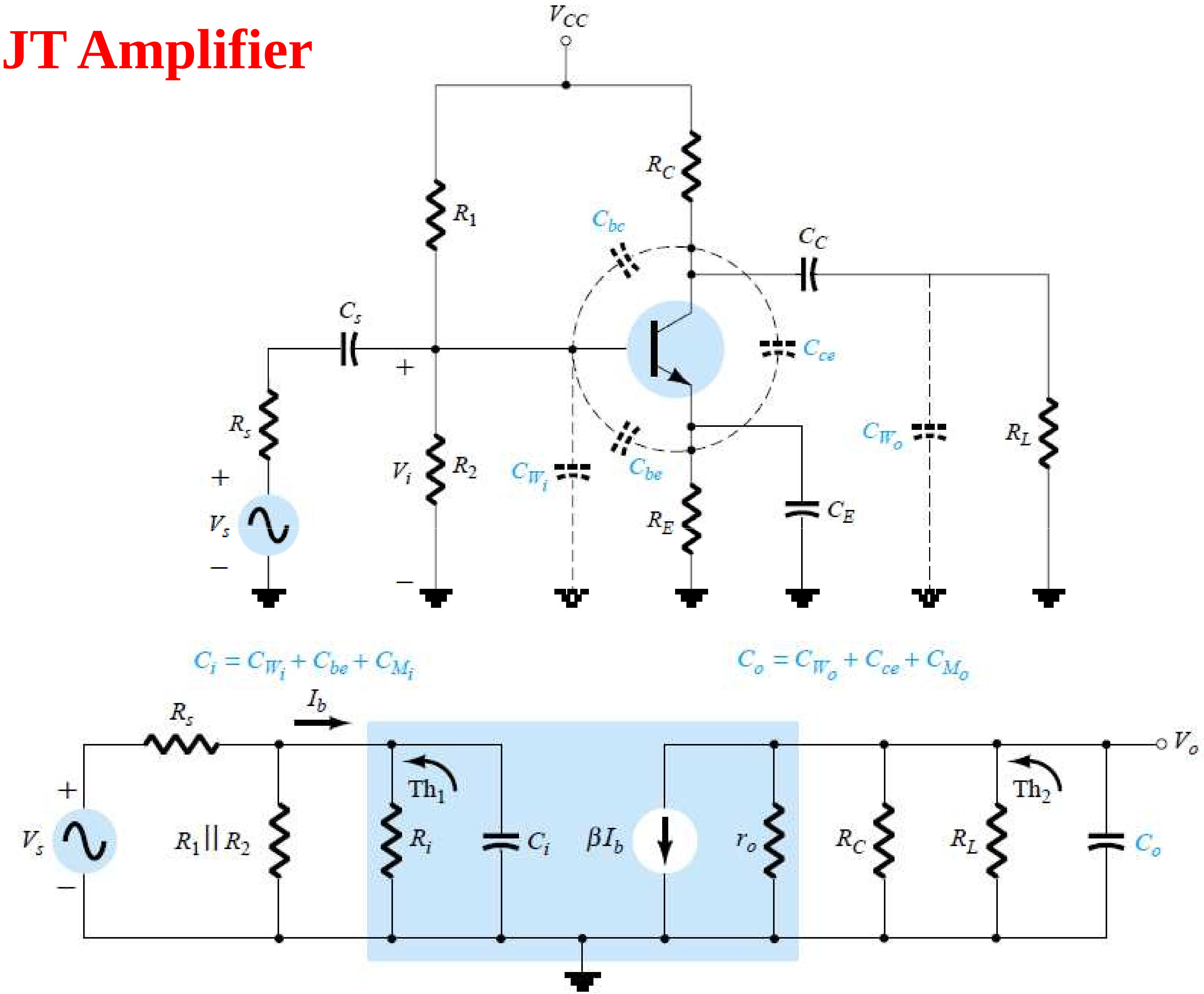
5. High-Frequency Response

You have seen how the coupling and bypass capacitors affect the voltage gain of an amplifier at lower frequencies where the reactances of the coupling and bypass capacitors are significant. In the midrange of an amplifier, the effects of the capacitors are minimal and can be neglected. If the frequency is increased sufficiently, a point is reached where the transistor's internal capacitances begin to have a significant effect on the gain.

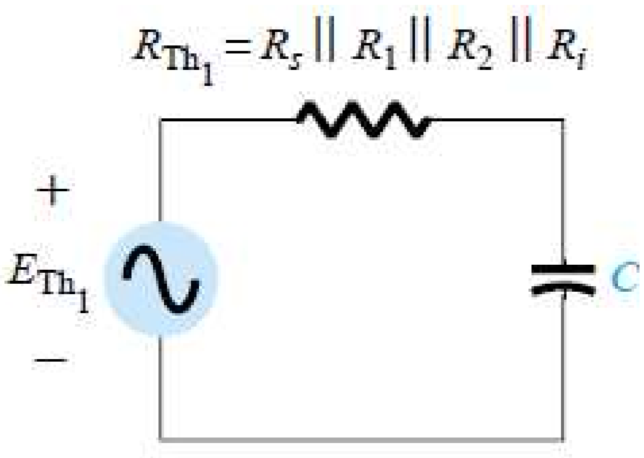
In the high-frequency region, the capacitive elements of importance are the interelectrode (between terminals) capacitances internal to the active device and the wiring capacitance between leads of the network. The large capacitors of the network that controlled the low-frequency response have all been replaced by their short-circuit equivalent due to their very low reactance levels.

For *inverting* amplifiers (phase shift of 180° between input and output resulting in a negative value for A_v), the input and output capacitance is increased by a capacitance level sensitive to the interelectrode capacitance between the input and output terminals of the device and the gain of the amplifier.

BJT Amplifier



The Thévenin equivalent circuit for the input network

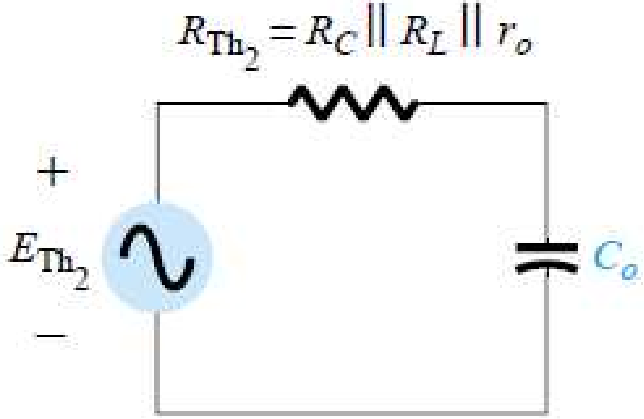


$$f_{Hi} = \frac{1}{2\pi R_{Th1} C_i}$$

$$R_{Th_1} = R_s \| R_1 \| R_2 \| R_i$$

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc}$$

The Thévenin equivalent circuit for the output network



$$f_{H_o} = \frac{1}{2\pi R_{Th_2} C_o}$$

$$R_{Th_2} = R_C \| R_L \| r_o$$

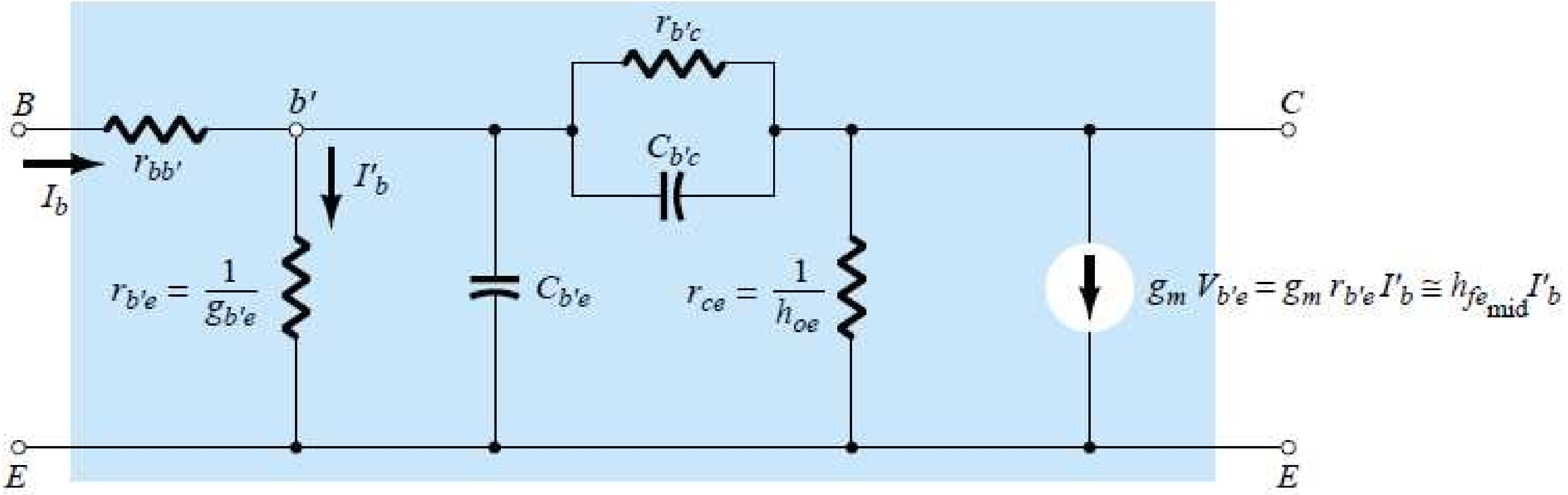
$$C_o = C_{W_o} + C_{ce} + C_{M_o}$$

h_{fe} (or β) Variation

The variation of h_{fe} (or β) with frequency will approach, with some degree of accuracy, the following relationship:

$$h_{fe} = \frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}$$

The only undefined quantity, f_β, is determined by a set of parameters employed in the *hybrid* π or *Giacoletto* model frequently applied to best represent the transistor in the high-frequency region.



$$f_\beta \text{ (sometimes appearing as } f_{h_{fe}}) = \frac{g_{b'e}}{2\pi(C_{b'e} + C_{b'c})}$$

or since the hybrid parameter h_{fe} is related to $g_{b'e}$ through $g_m = h_{fe_{\text{mid}}} g_{b'e}$,

$$f_\beta = \frac{1}{h_{fe_{\text{mid}}}} \frac{g_m}{2\pi(C_{b'e} + C_{b'c})}$$

Taking it a step further,

$$g_m = h_{fe_{\text{mid}}} g_{b'e} = h_{fe_{\text{mid}}} \frac{1}{r_{b'e}} \cong \frac{h_{fe_{\text{mid}}}}{h_{ie}} = \frac{\beta_{\text{mid}}}{\beta_{\text{mid}} r_e} = \frac{1}{r_e}$$

and using the approximations

$$C_{b'e} \cong C_{be} \qquad \text{and} \qquad C_{b'c} \cong C_{bc}$$

will result in the following form

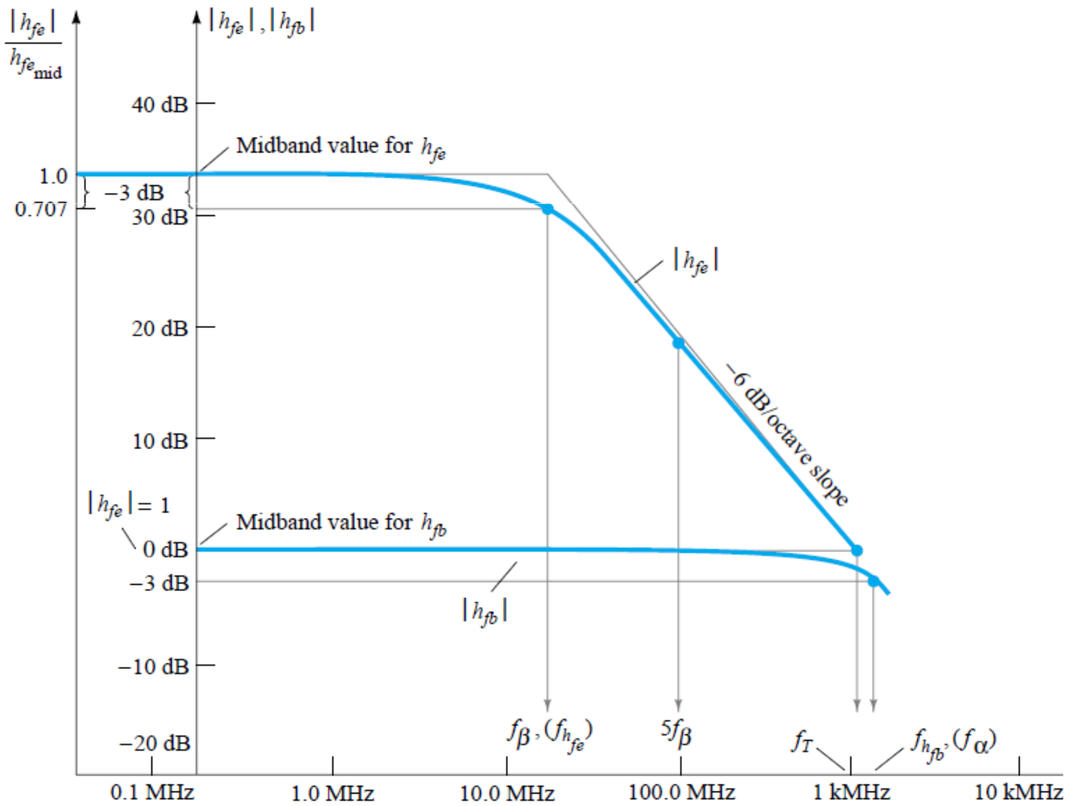
$$f_\beta \cong \frac{1}{2\pi\beta_{\text{mid}} r_e (C_{be} + C_{bc})}$$

f_β is a function of the bias conditions.

A quantity called the *gain–bandwidth product* is defined for the transistor by the condition

$$\left|\frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}\right| = 1$$

so that $$|h_{fe}|_{\text{dB}} = 20 \log_{10} \left|\frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}\right| = 20 \log_{10} 1 = 0 \text{ dB}$$



The frequency at which $|h_{fe}|_{dB} = 0$ dB is clearly indicated by f_T in Fig. above. The magnitude of h_{fe} at the defined condition point ($f_T \gg f_\beta$) is given by

$$\frac{h_{fe_{mid}}}{\sqrt{1+(f_T/f_\beta)^2}} \cong \frac{h_{fe_{mid}}}{f_T/f_\beta} = 1$$

so that

$$f_T \cong h_{fe_{mid}} \cdot \underbrace{f_\beta}_{(\cong \text{BW})} \quad \text{(gain–bandwidth product)}$$

or

$$f_T \cong \beta_{mid} f_\beta$$

Substituting equation for *fβ* gives

$$f_T \cong \beta_{mid} \frac{1}{2\pi\beta_{mid} r_e (C_{be} + C_{bc})}$$

and

$$f_T \cong \frac{1}{2\pi r_e (C_{be} + C_{bc})}$$

Example 6

For the following network with the same parameters as in Example 4 that is,

$R_s = 1\ \text{k}\Omega$, $R_1 = 40\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, $R_E = 2\ \text{k}\Omega$, $R_C = 4\ \text{k}\Omega$, $R_L = 2.2\ \text{k}\Omega$

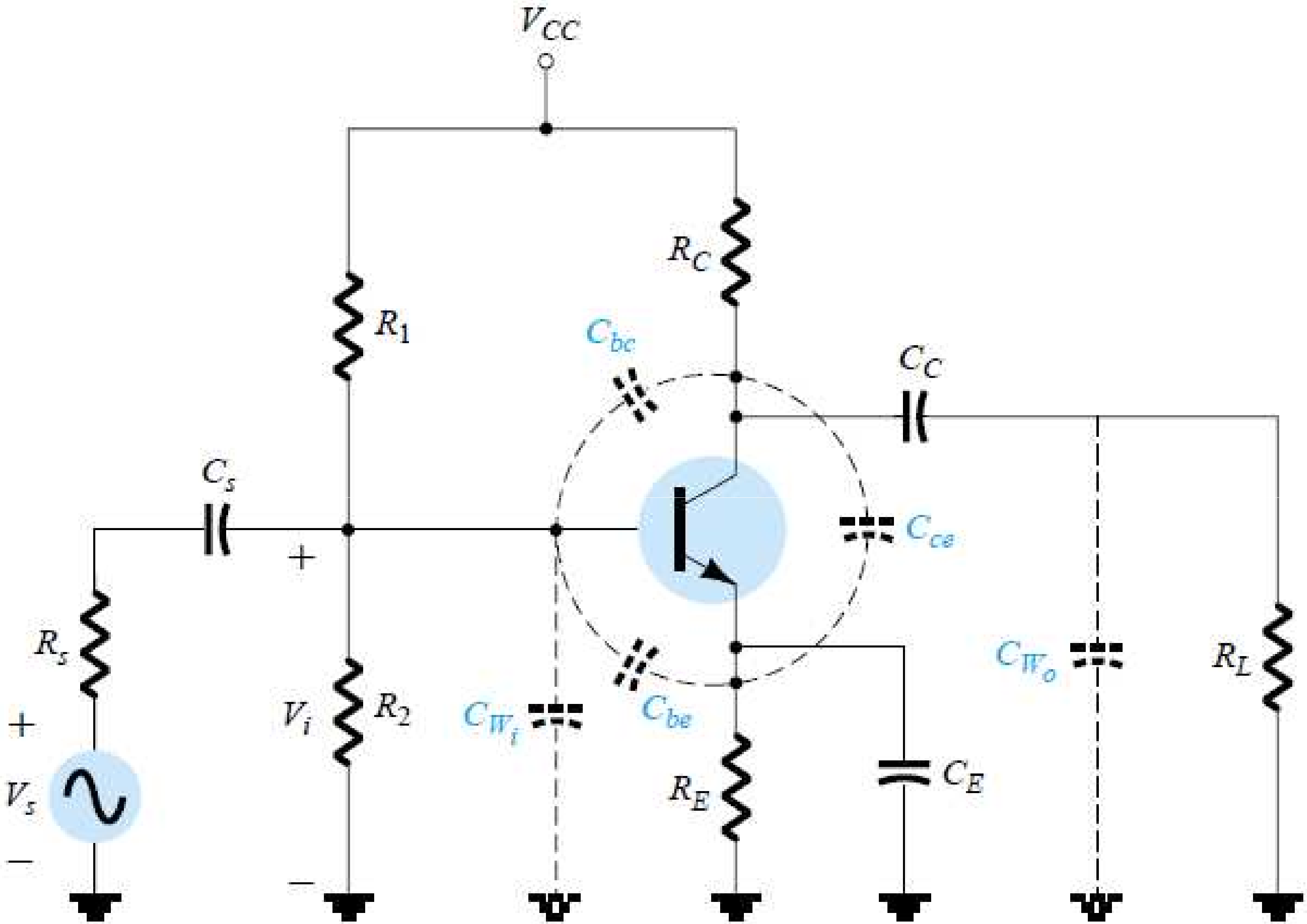
$C_s = 10\ \mu\text{F}$, $C_C = 1\ \mu\text{F}$, $C_E = 20\ \mu\text{F}$

$\beta = 100$, $r_o = \infty\ \Omega$, $V_{CC} = 20\ \text{V}$

with the addition of

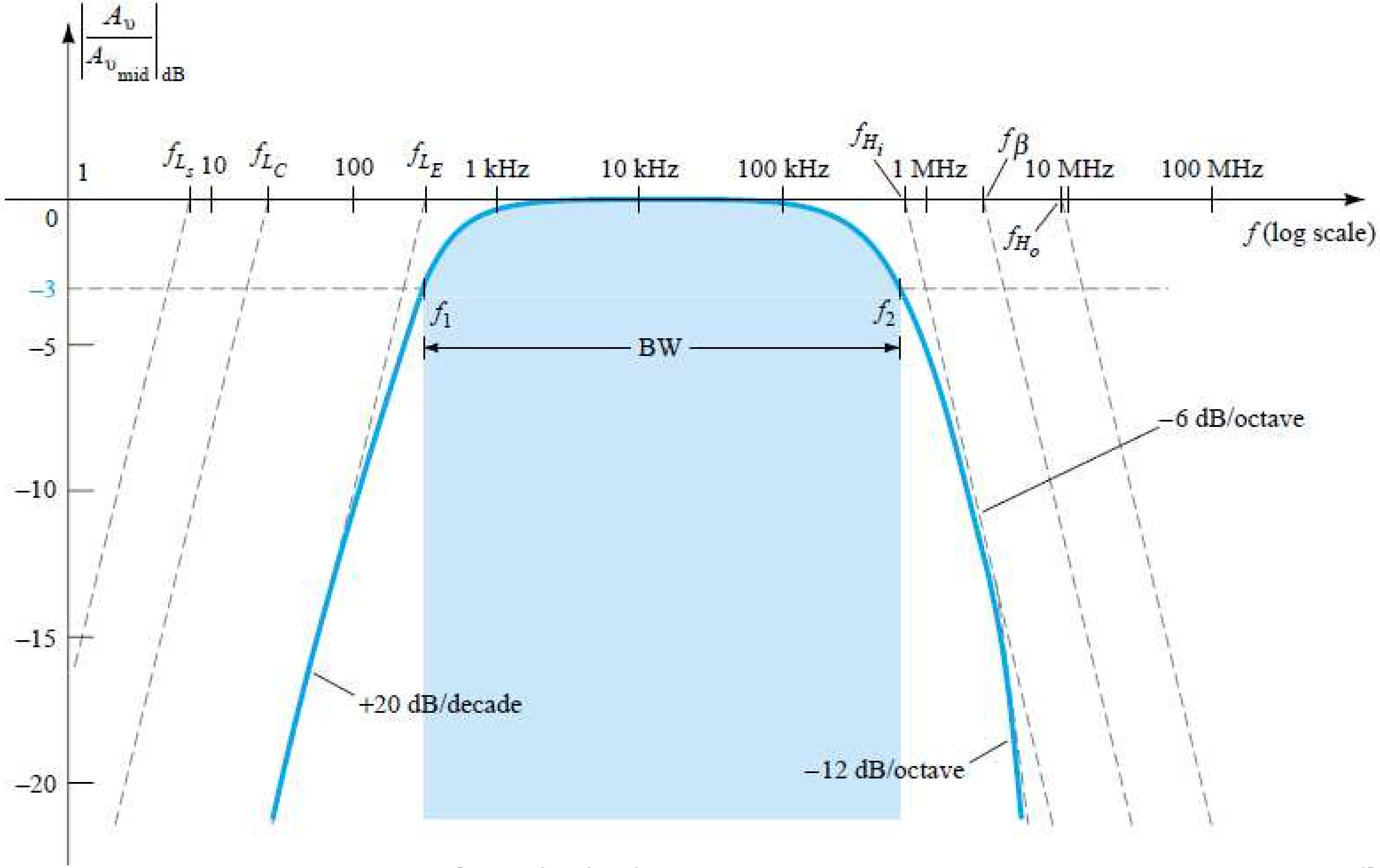
$$C_{be} = 36\ \text{pF},\ C_{bc} = 4\ \text{pF},\ C_{ce} = 1\ \text{pF},\ C_{W_i} = 6\ \text{pF},\ C_{W_o} = 8\ \text{pF}$$

(a) Determine f_{H_i} and f_{H_o}.
(b) Find f_β and f_T.
(c) Sketch the frequency response for the low- and high-frequency regions using the results of Example 11.9 and the results of parts (a) and (b).
(d) Obtain a **PROBE** response for the full frequency spectrum and compare with the results of part (c).


V_CC
R_C
R_1
C_bc
C_C
C_s
+
C_ce
R_s
C_W_o
R_L
+
V_i
R_2
C_W_i
C_be
V_s
R_E
C_E
−
−

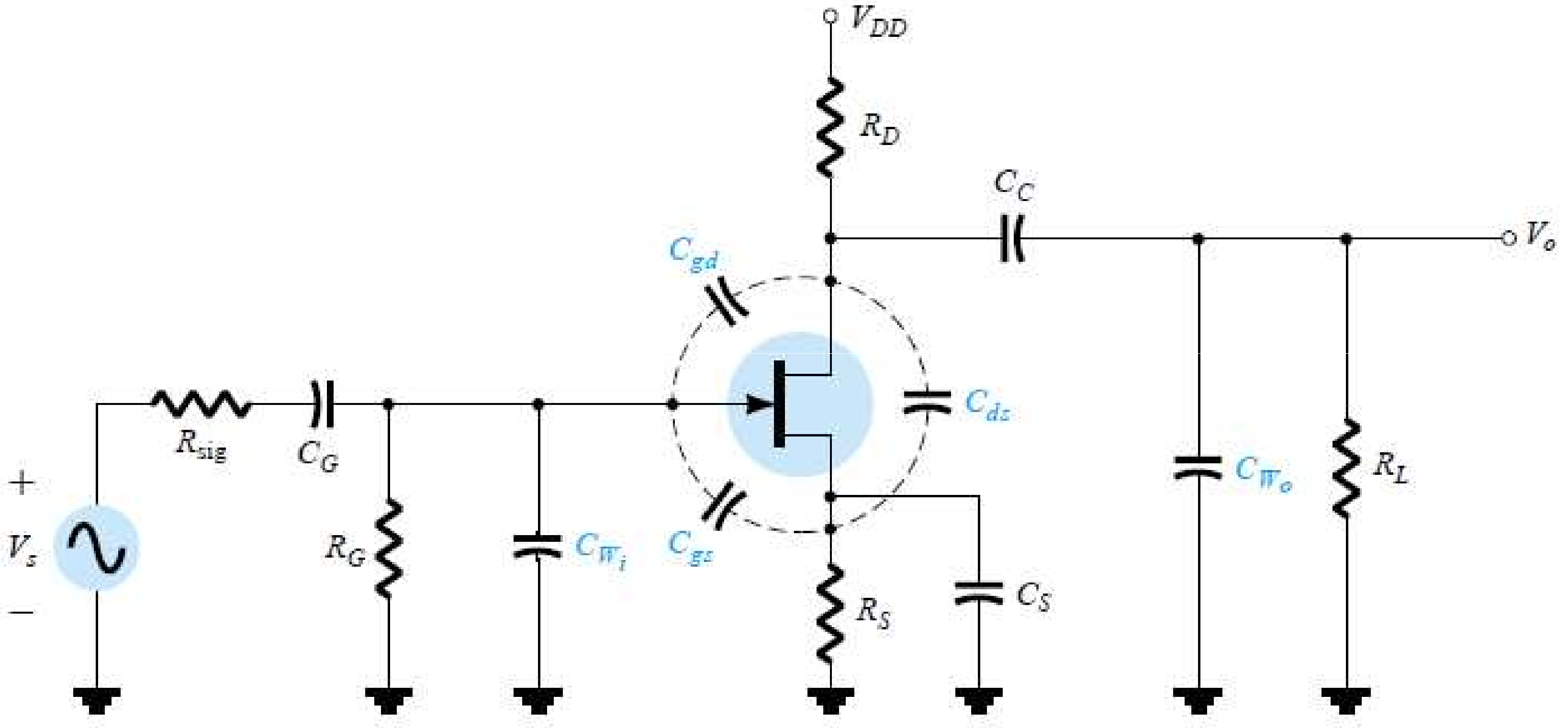
Solution

(c) See the figure. Both f_β and f_{H_o} will lower the upper cutoff frequency below the level determined by f_{H_i}. f_β is closer to f_{H_i} and therefore will have a greater impact than f_{H_o}. In any event, the bandwidth will be less than that defined solely by f_{H_i}. In fact, for the parameters of this network the upper cutoff frequency will be relatively close to 600 kHz.

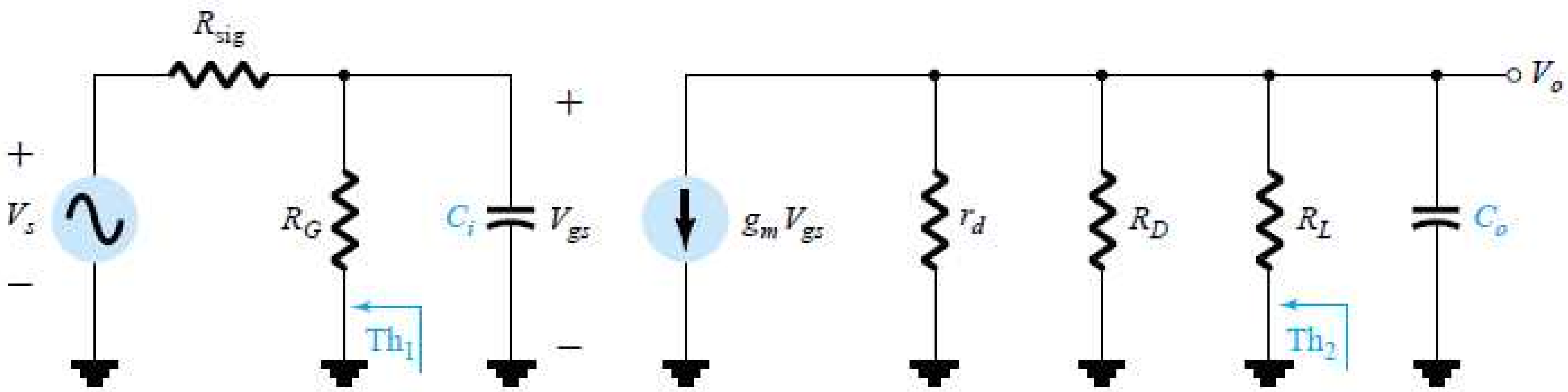


FET Amplifier

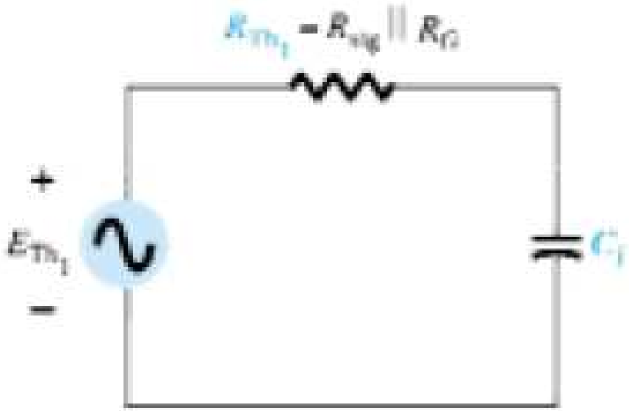
The analysis of the high-frequency response of the FET amplifier will proceed in a very similar manner to that encountered for the BJT amplifier. ,



The cutoff frequencies defined by the input and output circuits can be obtained by first finding the Thévenin equivalent circuits for each section



For the input circuit,



$$f_{H_i} = \frac{1}{2\pi R_{Th_1} C_i}$$

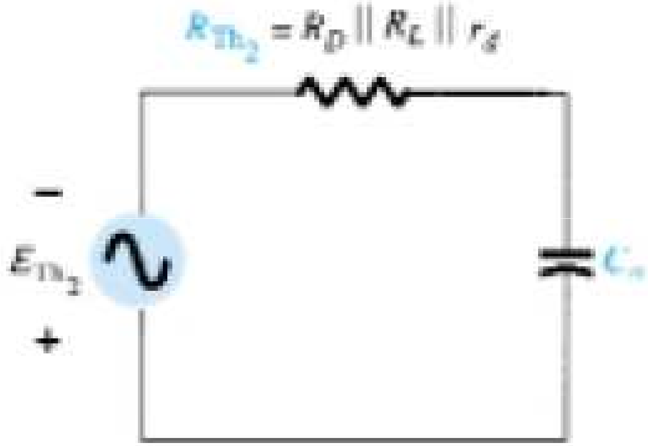
$$R_{Th_1} = R_{sig} \| R_G$$

$$C_i = C_{W_i} + C_{gs} + C_{M_i}$$

$$C_{M_i} = (1 - A_v) C_{gd}$$

and for the output circuit.

$$f_{H_o} = \frac{1}{2\pi R_{Th_2} C_o}$$



$$R_{Th_2} = R_D \| R_L \| r_d$$

$$C_o = C_{W_o} + C_{ds} + C_{M_o}$$

$$C_{M_o} = \left(1 - \frac{1}{Av}\right) C_{gd}$$

Example 7

(a) Determine the high cutoff frequencies for the following network using the same parameters as Example 5

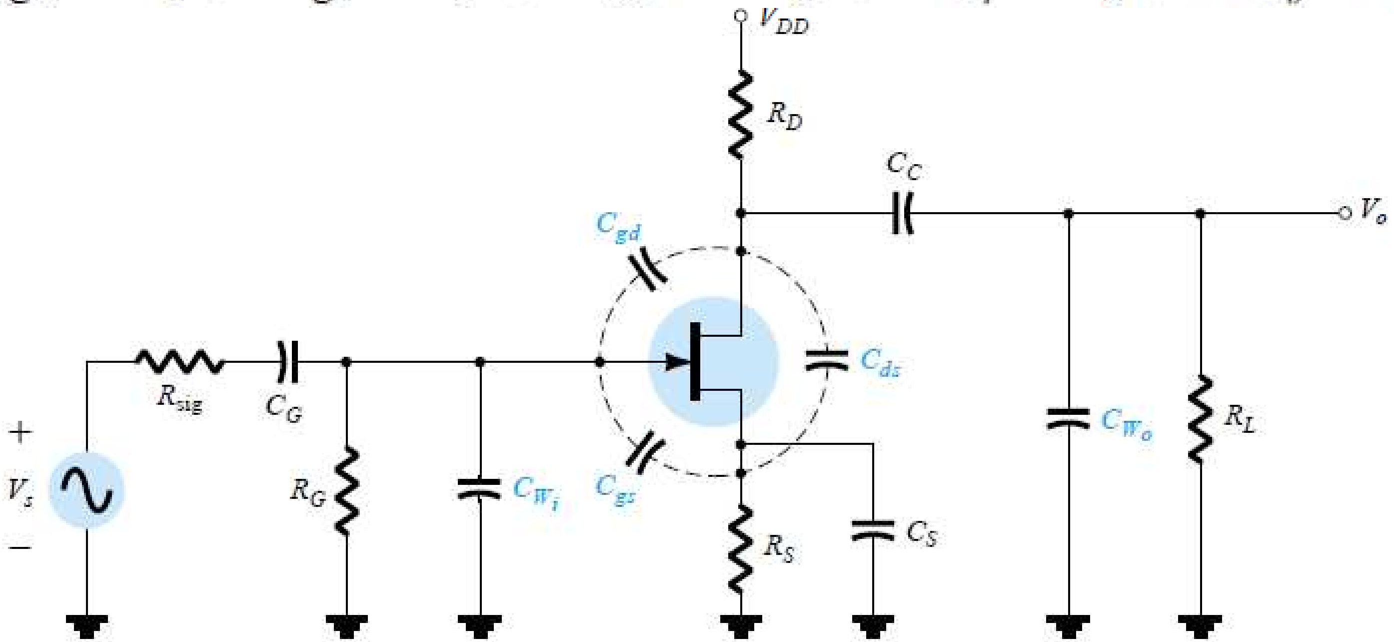
$$C_G = 0.01\ \mu\text{F}, \qquad C_C = 0.5\ \mu\text{F}, \qquad C_S = 2\ \mu\text{F}$$

$$R_{\text{sig}} = 10\ \text{k}\Omega, \quad R_G = 1\ \text{M}\Omega, \quad R_D = 4.7\ \text{k}\Omega, \quad R_S = 1\ \text{k}\Omega, \quad R_L = 2.2\ \text{k}\Omega$$

$$I_{DSS} = 8\ \text{mA}, \qquad V_P = -4\ \text{V}, \qquad r_d = \infty\ \Omega, \qquad V_{DD} = 20\ \text{V}$$

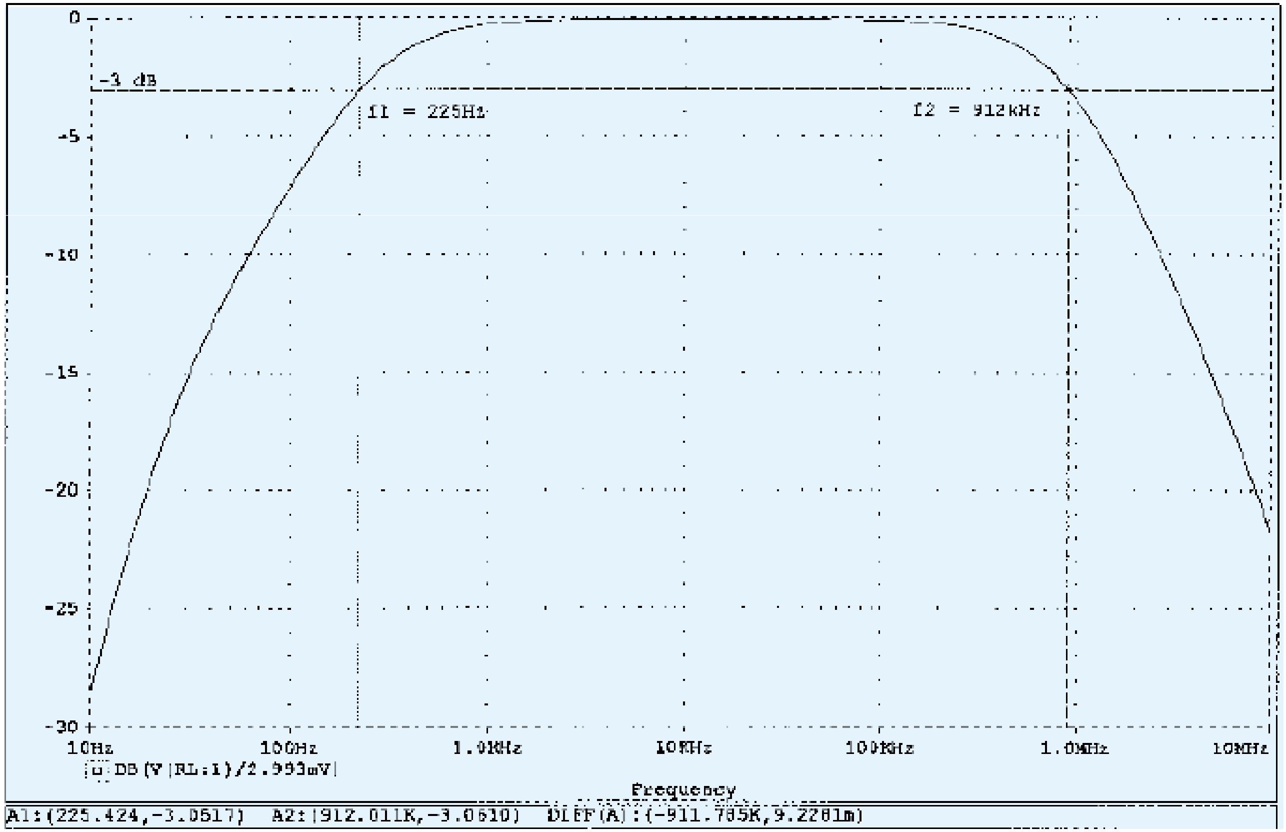
with the addition of

$$C_{gd} = 2\ \text{pF}, \quad C_{gs} = 4\ \text{pF}, \quad C_{ds} = 0.5\ \text{pF}, \quad C_{W_i} = 5\ \text{pF}, \quad C_{W_o} = 6\ \text{pF}$$



Solution

The results above clearly indicate that the input capacitance with its Miller effect capacitance will determine the upper cutoff frequency. This is typically the case due to the smaller value of C_{ds} and the resistance levels encountered in the output circuit.



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